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Tomography and Applications

Preface to the Special Issue

1. Introduction

This special issue of Fundamenta Informaticae is based on the 16-th edition of the Meeting on Tomography and Applications, that took place from May 2 to May 4, 2022 at the Department of Mathematics of Politecnico di Milano.

The meeting was dedicated to our dear friend Carla Peri (1957-2022), who was an active organizer of this event since its first edition in 2007. Also, this special issue is dedicated to her memory, recognizing her research, educational and organizing activities in the area of discrete tomography.

The Meeting on Tomography and Applications is by now a well-established international forum for the tomographic research community. Neuroimaging and network models have been included in the topics of the conference since its 11^{th} edition. Indeed, the area of neuroscience perfectly fits into the scope of the meeting, since it also considers advanced imaging techniques, such as diffusion MRI and PET, which are strongly related to inverse problems, contributing in this way to a multidisciplinary approach to image reconstruction.

As usual, also the focus of this 16^{th} edition was an interplay between the experimental research concerning X-ray tomography and the mathematical image reconstruction results. It offered a friendly and stimulating environment for international forum of researchers from various disciplines, including mathematics, computer science, engineering, physics, medicine, and biology.

After a rigorous refereeing process, four papers have been selected for this special issue. They offer valuable insights into both the theoretical and the application issues concerning tomographic research.

The paper by M. Ceko, L. Hajdu, and R.Tijdeman investigates situations where discrete tomography is utilized for data storage or transmission, resulting in a few inaccurate line sums.

The second paper, by Y. Gerard, deals with the reconstruction of convex lattice sets from their horizontal and/or vertical X-rays. The author presents three interesting results. He shows that the convex aggregation step used for the reconstruction of HV-convex polyominoes does not always provide a solution. Then, he proves that it is possible to reconstruct a digital convex lattice set from a single

X-ray in polynomial time. Finally, exploiting the same approach, he shows that the reconstruction of fat digital convex sets from their horizontal and vertical X-rays can also be solved in polynomial time.

The paper by C. Vincze, and A. Nagy belongs to the area of Geometric Tomography, and it deals with convergence results. It presents an overview of how taxicab distance mean functions and the theory of generalized conics can be exploited in geometric tomography for solving reconstruction problems from coordinate X-rays.

The fourth article, by D. Coluzzi, and G. Baselli, deals with a Bootstrapped Top-Down Approach to diffuse and localized functional dysconnectivity Schizophrenia. It outlines a multi-level robust approach for examining brain connectivity. The initial step involves analyzing global connectivity measures that can be explored with greater stability and robustness through bootstrapping. Subsequently, this approach is used to identify overall abnormalities within case-control groups.

The reminder of this preface consists of two parts: the first one concerns the contents of the invited talks presented at the meeting, while the second part provides an overview of the contributed talks.

2. Outline of the talks presented during theMeeting on Tomography and Applications, May 2-4, 2022, Mathematics Department, Politecnico di Milano.

The lecture given by Lama Tarsissi, the Ambassador of Committee for Women in Mathematics, UAE, was dedicated to the memory of Carla Peri.

The program of the meeting contained the following lectures.

INVITED LECTURES

- Jaques-Olivier Lachaud , Université Savoie Mont Blanc, France

  Alternative definitions for digital convexity.

  Abstract. This talk proposes full convexity as an alternative definition of digital convexity, which is valid in arbitrary dimension. It solves many problems related to its usual definitions, like possible non connectedness or non simple connectedness, while encompassing its desirable features. Fully convex sets are digitally convex, but are also connected and simply connected. They have a morphological characterisation, which induces a simple convexity test algorithm. Arithmetic planes are fully convex too. Full convexity implies local full convexity, hence it enables local shape analysis, with an unambiguous definition of convex, concave and planar points. We propose also a natural definition of tangent subsets to a digital set. It gives rise to the tangential cover in 2D, and to consistent extensions in arbitrary dimension. We present two applications of tangency: the first one is a simple algorithm for building a polygonal mesh from a set of digital points, with reversibility property, the second one is the definition and computation of shortest paths within digital sets. In a second part of the talk, we study the problem of building a fully convex hull. We propose an iterative operator for this purpose, which computes a fully convex enveloppe in finite time. We can even build a fully convex enveloppe within another fully convex set (a kind of relative convex hull). We show how it induces several natural digital polyhedral models, whose cells of different dimensions are all fully convex sets. As perspective to this work, we study the problem of fully convex set intersection, which is the last step toward a full digital analogue of continuous convexity.

- Toni Bohnlein, Weizmann Institute of Science

  Generalized Microscopic Image Reconstruction Problems.

  Abstract. We consider a generalization of the microscopic image reconstruction problem (MIR) where the task is to inspect a specimen represented as a collection of points typically organized on a grid in the plane. Assume that each point x has an associated physical value l(x), which we would like to determine. However, it might be that obtaining these values precisely (by what we call a surgical probe) is difficult, risky, or impossible. The alternative is to employ aggregate measuring techniques (such as CT or MRI), whereby each measurement is taken over a larger window, and the exact values at each point are subsequently extracted by computational methods. We extend the MIR framework in a number of ways. First, we study a generalized setting, where the inspected object is represented by an arbitrary graph G and the vector l in Rn assigns a value l(v) to each node v. A probe P(v) centred at a node v will capture a window encompassing its entire neighbourhood N[v], i.e., P(v) sums l(u), for each u in N[v]. Additionally, we analyze a weighted variation in which a node v may have an aggregation coefficient w(v), namely P(v) sums l(u) and w(v) l(v), for each u in N[v]. We give a criterion for the graphs for which the extended MIR problem can be solved by extracting the vector l from the collection of probes, P = P(v) for each v in V. We then consider cases where such reconstruction is impossible (namely, graphs G for which the probe vector P is inconclusive, in the sense that there may be more than one vector l yielding P). Assume that surgical probes (whose outcome at node v is the exact value of $l(v)$) are technically available, yet are expensive or risky, and must be used sparingly. We show that in such cases, it may still be possible to achieve reconstruction based on a combination of a collection of ordinary (aggregate) probes together with a suitable set of surgical probes. We aim at identifying the minimum number of surgical probes necessary for a unique reconstruction, depending on the graph topology. This is referred to as the Minimum Surgical Probing problem (MSP). Besides providing a solution for the above problems for arbitrary graphs, we also explore the range of possible behaviours of the (MSP) problem by determining the number of surgical probes necessary in certain specific graph families including grid graphs.

- Gabriele Fici, Universitá di Palermo, Italy

  Minimal Forbidden Words and Digital Lines.

  Abstract. Minimal Forbidden Words (a.k.a. Minimal Absent Words) are a powerful tool for dealing with words and languages (set of words). Given a language L that is closed under taking factors (i.e., a factorial language) the set of Minimal Forbidden Words of L is the set of words that do not belong to L, but whose proper factors belong to L. There is a bijection between factorial languages and their sets of MFWs. Hence, one can study the combinatorial properties of a language by looking at its set of Minimal Forbidden Words. In the context of digital geometry, one is often interested in binary words approximating Euclidean segments in the plane (balanced words). Among them, Christoffel words play a central role. A Christoffel word is a balanced word with the Lyndon property, i.e., it is lexicographically smaller than all its nontrivial rotations. Another interesting class is that of digitally convex words, i.e., binary words approximating convex lines in the plane. These can be defined as those words whose Lyndon factorization consist of balanced (hence Christoffel) words. We discuss the set of MFWs of balanced words and of digitally convex words.

- Sara Brunetti, Universitá di Siena, Italy

  How to rebuild a binary image from its multi-level description based on generalized salient pixels

  Abstract. In Discrete Tomography, several classes of binary images presenting different kind of convexity have been considered for their reconstruction from projections.In Digital Image Analysis convexity estimators are among the most important shape descriptors. Shape feature extraction and representation plays an important role in many categories of applications like for example shape retrieval, shape recognition and classification, shape approximation and simplification, and so on. In this talk, we present a multi-level description of a binary image based on a special kind of convexity. In particular, the so called generalized salient pixels provides a decomposition of the image into Q-convex hulls at different levels and they are stored in a matrix called, GS-matrix (where GS stands for Generalized Salient). Therefore, there is a one-to-one correspondence between the binary image and its GS-matrix. We show how to build the GS-matrix from the binary image and vice versa how to rebuild the binary image from its GS-matrix. Then, we play with GS-matrices to see how changes can modify the rebuild images. In Mathematical Morphology, a wide range of operators permits to process images for edge detection, noise removal, image enhancement and image segmentation, to mention some common usage. Among them, the two most basic operations are erosion and dilation. Virtually all other mathematical morphology operators can be defined in terms of combinations of erosion and dilation along with set operators such as intersection and union. We start by defining a new "erosion" operation based on the interaction of GS-matrix binary image and we investigate some consequences.

- Cécile Bordier, Centre Hospitalier Universitaire de Lille, France

  Brain connectivity in MRI: methods, interpretation and needs.

  Abstract. After an introduction to MRI and its various acquisitions and the different schools of thought that have led us to connectivity, I will present methods to perform connectivity analysis. From ICA to graph theory via statistical permutation methods, I will try to give you a general idea of the possible methods and the potential problems encountered. I will finish with concrete applications on patient populations before reviewing the current needs and the methods under development.

- Davide Coluzzi, Politecnico di Milano, Italy

  Brain Connectivity through Graph Theory: SPIDER-NET a New Tool to Explore Sub-Networks.

  Abstract. Brain connectomics consists in the modelling of human brain as a network, mathematically represented as a numerical connectivity matrix. It is represented by collection of nodes (brain regions) and links (connections), describing any kind of relationship between pairs of nodes, at different scales.

The brain regions are strongly connected through neuroanatomical white matter (WM) pathways which can be quantified by the number of streamlines derived with deterministic WM tractography or other metrics. In parallel to structural connectivity (SC), synchronous and asynchronous activity of specific brain regions results in related complex cognitive functions, which can be investigated in terms of functional connectivity (FC). Conversely, FC represents the magnitude of temporal correlations between the blood oxygenation level-dependent (BOLD) time series produced by pairs of brain regions. In both cases, the resulting networks determine a complex system involving several interconnected nodes relevant to the partition of cortical areas and sub-cortical structures which is difficult to be interpreted. To overcome this limitation, circular representation of connectivity by “connectograms” is currently used via open-source tools, which, however, lack user-friendly interfaces and options to explore specific sub-networks.

A complete whole-brain network can be indeed made of thousands of links, and it is well-known that this large-scale network is associated with high-level cognitive functions. However, the brain is composed of several interacting lower-scale sub-networks, which are characterized by distinct patterns of brain activation, identifying specific domains of behavior and cognition. Therefore, a common practice in explorative studies of brain connectivity is the extraction of valuable sub-networks referenced by the domain knowledge. Focusing on sub-networks, analyzed both qualitatively, by a reduced connectogram, and quantitively, by local and global subgraph indexes, can lead to easier data interpretation driven by the addressed physiological and/or pathological problem. In this context, we developed SPIDER-NET (Software Package Ideal for Deriving Enhanced Representations of brain NETworks), an easy-to-use, flexible, and interactive tool for connectograms generation and sub-network exploration.

The potential impact of the tool was tested on pilot cases.

- Niccoló di Marco, Universitá di Firenze, Italy

  The null label problem and its relation to the 2-intersection graph.

  Abstract. A 3-uniform hypergraph H consists of a set V of vertices, and a subset E of triples from V, called set of edges. Let a null labeling be an assignment of +1 or -1 to the triples such that each vertex has a signed degree equal to zero. If a null labeling exists, we say that the hypergraph is null. Assume that the degree of every vertex of H is even. Then the Null Labeling Problem consists in determining whether H has a null labeling. It is remarkable that null hypergraphs arise considering two hypergraphs with the same degree sequence. In particular, the symmetric differences of these two hypergraphs give a new hypergraph that is null. From a discrete tomography point of view, null hypergraphs arise from matrices with the same projections, i.e., solutions of the same reconstruction problem. Therefore they allow modeling of switching components, a common notion in this field of research.

  Although the general problem is NP-complete, the subclasses where the problem is polynomially solvable turn out to be interesting. We define the notion of 2-intersection graph related to a 3-uniform hypergraph and prove that if it is Hamiltonian, then the related 3-hypergraph has a null labeling. Then we investigate some structural properties of 2-intersection graphs. More specifically, we consider the following problem: for a given graph G is it possible to find a 3-hypergraph such that its 2-intersection graph is G? If it is possible, then we say that G is reconstructable or equivalently, that it has the 2-intersection property. Its easy to see that the answer to this question is relatively straightforward for some classes. However, we prove that the problem in its general form is NP-Complete.

- Laurent Vuillon, Université Savoie Mont Blanc, France

  From fibers to brain diseases.

  Abstract. In this talk, the first part presents how to construct biological fibers from proteins and how to build a model using tiling theory and discrete geometry. The second second part is devoted to the description of the fiber formation in Parkinsons and Alzheimers diseases. In the third part, we describe coarse-grain simulations where proteins are modelled as semi-flexible polymers with multifunctional binding sites in order to obtain a model of biomolecular condensates and to show some different behaviors (fluid versus rigid phase) according to various parameters (affinity, concentration,). In fact, understanding the transition between fluid phase to rigid phase is important because this phenomenon is linked to certain neurodegenerative diseases. This is a joint work with Julian C. Shillcock (EPFL), Claire Lesieur (Ampére and IXXI), Clément Lagisquet (USMB), Jérémy Alexandre (EPFL) and John H. Ipsen (University of Souththern Denmark).

- Roberto Fedele, Politecnico di Milano, Italy

  A furnace for in situ wettability experiments at high temperatures under X-ray Microtomography.

  Abstract. In this study, we analyzed the problem of a compact furnace, to be used for in situ experiments in a cone-beam X-ray microtomography commercial system. The design process was accomplished and outlined through its main steps, until the realization of a prototype. The furnace was conceived to carry out wettability experiments at temperatures up to 700 C and under inert atmosphere on sessile droplets of a molten metal alloy, with a few millimeters diameter, posed on a thin ceramic substrate. X-ray imaging of the molten droplet is expected to allow for an accurate threedimensional reconstruction of the droplet profile and a robust estimation of the related quantities (such as the contact angle and the surface tension) utilized for the assessment of metal-ceramic joints by brazing. The challenges faced during this project, mostly related to the constraints of the setup, and the novel implemented solutions are discussed, supported by analytical and numerical tools, in terms of interaction of X-rays with matter, geometry and working principle, heat transfer and insulation, and material selection

- Clément Lagisquet, Université Savoie Mont Blanc, France

  Graph theory: From simulation to information.

  Abstract. This talk, which follows "From fibers to brain diseases", presents the techniques used to get meaningful informations from the data. The first will be how the graph (or multigraph) is constructed, the second is the link between clustering clustering (CC) and crowdfulness, and the third is the introduction of a new measure: the fluidity of the network, to understand how much the polymers flows in it.

- Stefania Petra, Heidelberg University, Germany

  Geometric Multilevel Optimization for Discrete Tomography

  Abstract. In this talk, I will present a geometric multilevel optimization approach to the reconstruction problem in discrete tomography based on undersampled projection data that is relevant to a range of applications and naturally leads to a hierarchy of models of varying discretization levels. Multilevel optimization is employed in order to take advantage of this hierarchy: while working at the fine level the search direction is computed based on a coarse model which speeds up tomographic reconstruction. We import some concepts of information geometry to the n-orthotope and propose a smoothing operator that uses only first-order information and incorporates constraints smoothly. We show that the proposed algorithm is well suited to the ill-posed reconstruction problem in discrete tomography and demonstrate its efficiency on several large-scale examples.

- Csaba Vincze, University of Debrecen, Hungary *Distance-mean functions and their geometric applications.*

  Abstract. A distance-mean function measures the average distance of points between the elements of a given subset K in the space. The level sets of a distance-mean function are called generalized conics with elements in K as focal points. The most important discrete examples are polyellipses (polyellipsoids) as the level sets of a function measuring the arithmetic mean of distances from finitely many focal points, i.e, the distance sum must be constant for the points of a polyellipse (polyellipsoid). Polynomial lemniscates can also be given as the level sets of a function measuring the geometric mean of distances from finitely many focal points, i.e, the distance product must be constant for the points of a polynomial lemniscate. We consider a lot of generalizations as well. Instead of formulating a basic definition to cover the wide range of possibilities we are going to present some applications in convex geometry (bisection of bodies by coordinate hyperplanes) and geometric tomography (reconstruction of planar bodies from the coordinate X-rays by the taxicab distance-mean function and some related results).

CONTRIBUTED TALKS

- Shi Jiayang, Leiden University, Netherlands

  Multi-stage Deep Learning Denoising for Computed Tomography.

  Abstract. Computed Tomography (CT) is a powerful tool in various areas. Recently, low-dose CT has drawn a lot of attention, because the high radiation dose in normal CT could be harmful to objects or patients. Reducing the projection numbers is also becoming a popular measure to improve the scanning speed. However, reconstructions from low-dose or/and limited projections typically suffer from high noise. Recently, deep learning has been proposed as an effective denoising technique for CT. However, when the noise level is high or when specific artifacts like ring or zinger artifacts occur, current deep learning approaches can yield suboptimal results. In

this talk, we propose a deep learning-based multi-stage denoising algorithm. In our algorithm, the denoising takes place subsequently in the projection domain, the sinogram domain, and the reconstruction domain, in a supervised manner. At each stage, both the denoised output from the previous stages and the raw noisy data are utilized for denoising. Experiments show that our algorithm is effective in denoising highly noisy data with additional ring or zinger artifacts.

- Giulia Palma, Universitá di Siena, Italy

  A Geometric Approach to Coalition Resilient Outcomes in Social Graphs modeled by the Max k-Cut Game.

  Abstract. We investigate strong Nash equilibria in the max k-cut game, where we are given an undirected unweighted graph together with a set of k colors. Vertices represent players and edges their mutual interests. By selecting a color players induce a coloring. Each player v has the set of k colors as strategy set, and its payoff is the number of neighbors of v that has chosen a different color. The max k-cut game has been extensively studied due to the great interest for its real-world applications with selfish agents. Very little is known about the existence of strong equilibria in such game. In this work we make some progress in the understanding of it, proving that there do not exist any subsets of nodes able to increase their own payoffs simultaneously. Instead of formulating a basic approach based only on the notions and tools of game theory, we split the nodes of the graph into three subsets (the coalition itself, the coalition boundary and the nodes without relationship to the coalition) and we use a novel approach based on discrete geometry and algorithms on graphs to study the properties of the adjacency matrix of the graph and obtain significant information on the coalition and its boundary.

- Fabrizio Pizzagalli, Universitá degli Studi di Torino, Italy

  Investigating genetic factors driving brain morphology through Neuroimaging studies.

  ABSTRACT Imaging genetics aims to identify genetic variants associated with the structure and function of the human brain. Recent studies were able to identify and replicate common genetic variants influencing human brain structure as measured through magnetic resonance imaging (MRI). In this talk, the impact of imaging genetic studies will be presented as one of the methods for understanding the causal chain from genetic variant to increased risk for disorder.

- Csaba Olasz, Szeged University, Hungary

  U-net based deep neural networks for transmission tomography.

  Abstract. The fusion of computer tomography and deep learning is an effective way of achieving improved image quality and artifact reduction in reconstructed images. The literature provides a variety of options for the combination of computer tomography and deep learning methods. Most of the time researchers applied deep learning as a pre- or post-processing tools before- or after the reconstruction, while others are interested in performing the reconstruction with the neural network. In this presentation, we focus on the U-net based combination of the neural networks and computer tomography. We compared two of our own U-net based neural network architectures to three other U-net based approaches. In the case of our own architectures the image reconstruction step is located inside the neural networks, which allows the network to be trained by taking the mathematical model of the projections into account. This strong connection enables us to enhance the projection data and the reconstructed image together. We tested the methods on two data sets. The data sets contain physically correct simulated data and they show strong signs of beam hardening and electrical noise. We also performed a numerical evaluation of the neural networks on the reconstructed images according to three error measurements and provided a scoring system of the methods derived from the three measures. Our experimental results demonstrate that the reconstruction step used in skip connections in deep neural networks improves the quality of the reconstructions.

- Marco Seracini - University of Bologna, Italy

  Inpainting in discrete Sobolev spaces: the subtle bond between content and structure.

  Abstract. In the literature, the so called inpainting problem consists in filling zones of missing information in an image, i.e., in a two-dimensional discrete set. To have satisfactory results, the inpainted area has to be not eye-distinguishable from the rest of the given image by a human observer. Different solutions have been proposed to tackle the problem: on one hand, PDE based approaches, on the other hand, exemplar-based ones. Starting from this last kind of techniques we consider discrete Sobolev spaces to formulate a new functional such that it's minimization provides good results in inpainting. Moreover, we extend the work of Criminisi et al. about structuring a new priority index. Our results show an effective uncertainty reduction in the choice of the missing points, with a consequent improvement in the quality of the final reconstructions.

- Mariusz Rzasa - Opole University of Technology, Poland

  Image reconstruction method for a discrete optical tomograph.

  Abstract. Optical tomography belongs to the area of absorption tomography, for which the natural signal at the output of the sensor is an analog signal. In some applications it is advisable to use appropriate discretization systems. As a result of the operation of such systems, a discrete measurement signal is obtained. This issue is discussed by using the example of a discrete optical tomograph for measuring moving gas bubbles. The presentation shows the structure of the tomograph. The prototype solution uses a light beam in the visible light range. To measure the intensity of light radiation, fiber optic detectors mounted in two perpendicular sensors were used. The small number of projections makes the reconstruction process more difficult. For this reason, a dedicated image reconstruction algorithm based on the Kohonen neural network has been proposed. Physical phenomena occurring on the border of the liquid and gas phases, which are the main cause of the change in the intensity of light reaching the detector, are discussed. Due to the fact that the shapes of the gas bubbles can differ significantly from each other and the number of projections is limited, the reconstruction of the image of the bubbles with the space was approximated by means of ellipses. Such an approximation is often sufficient to determine the parameters of mass transfer for the purposes of technological control of aeration and oxygenation processes.

- Lama Tarsissi- Ambassador of Committee for Women in Mathematics, UAE

  Women and mathematics. "Thank you Carla".

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